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Analytical Solutions for Capacitance of a Semi-Cylindrical Capacitive Sensor

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Abstract. We report the calculation of the capacitance for a Semi-Cylindrical Capacitive Sensor (SCS). In this study, we use conformal mapping to solve the SCS problem in the curve space. The SCS problem can be solved as parallel plates capacitors in the curved space. Based on our calculation, the capacitance of the SCS depends on both the area of the SCS plates and the dielectric materials. The capacitance of the SCS increase related to increasing the wide of SCS plates, in which the SCS can hold more charge. Otherwise, the capacitance of the SCS decrease related to decreasing the wide of SCS plates. The capacitance of the SCS increases along with inserting dielectric materials. The dielectric materials reduce the net electric field, in which to restore this conditions more charges are needed.

INTRODUCTION

A capacitive sensor has been used for many detection applications and measurements. Generally, the coaxial cylinder capacitive sensors (CCS) have been widely used for capacitive sensing [1]. The CCS has been used for measuring rotating devices such as in the case of a rotor. The CCS can detect the radial and the axial misalignment of rotor [2-4]. The CCS have also been used for measuring dielectric properties of materials. The applications of this study are for the steam wetness measurement based on the capacitive sensors and for the liquid level measurement for conductive liquid [5,6]. Previously, a theoretical study has been reported and describe a nonlinear analysis of the CCS [7]. Besides the CCS as capacitive sensors, the another type is the semi-cylindrical capacitive sensors (SCS). The SCS could be a promising candidate used for flow rate measurement of fluid. It can be implemented for medical instrumentation system such as intravenous drip [1].

The theoretical study of the capacitance of the SCS has been reported. The SCS can be approximated by the sum of a piece of the semi-parallel capacitors. In this case, numerical methods are needed to solve this approximation [1,8]. Recently, theoretical study of the curved patch capacitors (CPC) has been reported for a general case of the SCS. The CPC consists of two electrodes in which one of them can be driven to follow the cylindrical curve of the capacitors. The calculations of the capacitance of CPC has been included in this work.The calculation involves the completions of polynomials [9]. In this paper, we report analytical solutions of the capacitance from the SCS. The conformal mapping is selected instead of the polynomials methods. The conformal is an easier method than the polynomials, and it gives the exact solutions of the capacitance. The conformal mapping is used to mapped the SCS from the real space into the curve space. We use finite cylindrical isotropic dielectric rods and having the same radius of the SCS as material under test. The problems of the SCS with or without dielectric materials can be solved easily as parallel plate capacitors in the curve space.

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CALCULATION METHODS

The conformal mapping is used to convert the SCS problem from the real space into the problem of the parallel plate capacitors in the curve space. Figure 1 (a and b) show the cross section of the SCS without dielectric materials in the real space and in the curve space respectively. Meanwhile, Fig. 1 (c and d) describe the cross section of the SCS with inserted dielectric materials in the real spaces and in the curve spaced respectively. The curve space consists of two axes in which related to real axis and defined as follow [10]:

$$w = p + iq = f(x, y) + ig(x, y), \tag{1}$$

Where p and q are an axis in the curve space respectively. The connection of p axis related to the real space is given by [10]:

$$p = \arctan\left[\frac{2ay}{r^2 - (x^2 + y^2)}\right] \tag{2}$$

Where (x, y) are related to positions on the real axis and r is a radius of the capacitor. The cylindrical curve of the SCS can be expressed in equation (2) as $r^2 = x^2 + y^2$, so that equation (2) can be written as follow [10]:

$$p = \pm\frac{\pi}{2} \tag{3}$$

The connection of q axis related to the real space can be expressed as:

$$q = \ln\left[\frac{\sin\theta}{(1 - \cos\theta)}\right] \tag{4}$$

where θ is related to the gap between the edges of the SCS in which shown in Fig.1.

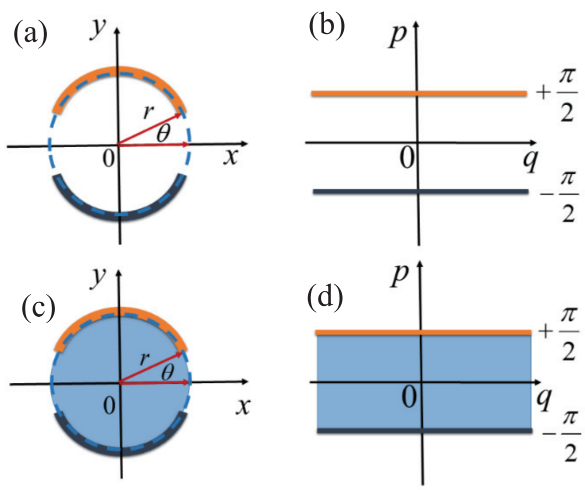

FIGURE 1. (a) and (b) the SCS without dielectric materials in the real spaces and curved space respectively, (c) and (d) the SCS with dielectric materials in the real spaces and curve spaced respectively.

RESULTS AND DISCUSSION

The capacitance of the SCS can be calculated as a parallel plate capacitor in the curved space. The capacitance of parallel plates capacitor defined as below:

$$C = \frac{A\varepsilon_0}{d}, \tag{5}$$

Where ε_0 is permittivity in free space $\left(8.85\times10^{-12}\ \mathrm{C^2/N\,m^2}\right)$, A is related to a surface area of the plates of the capacitor and d is the distance between the plates. The area of the SCS plates in the curved space can be calculated from equation (4). It can be expressed as:

$$A = 2z\ln\left[\frac{\sin\theta}{(1-\cos\theta)}\right], \tag{6}$$

Where z is the length of the SCS. The distance between the plates of the SCS from equation (4) can be expressed as π. From this analogy the capacitance of the SCS can be expressed as:

$$C_0 = \frac{2z\varepsilon_0}{\pi}\ln\left[\frac{\sin\theta}{(1-\cos\theta)}\right], \tag{7}$$

For the case of the SCS with dielectric material, the capacitance is given by

$$C = \kappa C_0 = \frac{2\kappa z\varepsilon_0}{\pi}\ln\left[\frac{\sin\theta}{(1-\cos\theta)}\right], \tag{8}$$

where κ is a dielectric constant of the testing materials.

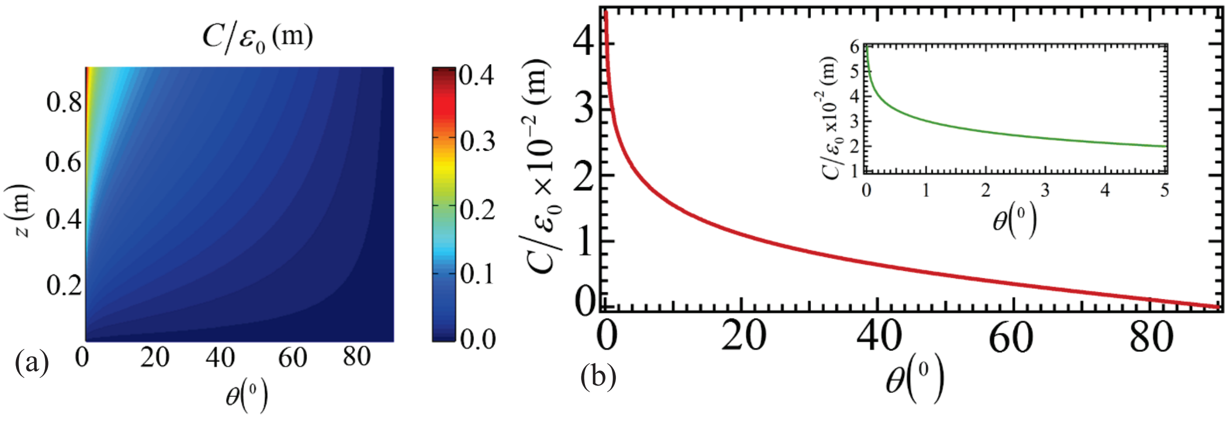

FIGURE 2. (a) The Capacitance of the SCS related with the variation of θ and z with $\kappa = 1$. (b) The Capacitance of the SCS related with the variation of θ from 0 to 90 degrees with $\kappa = 1$, the inset picture shows that the Capacitance of the SCS for θ between 0 to 5 degrees.

Figure (2) and Figure (3) shows the calculation of the capacitance of the SCS using equation (8). Figure (2a) show the calculation of the capacitance including the variation of θ and z, in which κ sets as one related to vacuum inside the cavity of the SCS[11]. Figure (2b) shows the capacitance of the SCS including the variation of θ from 0 degree until 90 degree. We show that the increase of capacitance of the SCS along with decreasing the value of θ and increasing the value of z. The increasing of the capacitance of the SCS is related to increasing of the wide of the SCS plates, in which the SCS can hold more charge. Otherwise, the capacitance of the SCS decreases along with increasing of θ. The capacitance of the SCS nearly zero when θ closed to 90 degree. The increasing value of θ will reduce the area of the SCS plates in which that the SCS can not hold more charge.

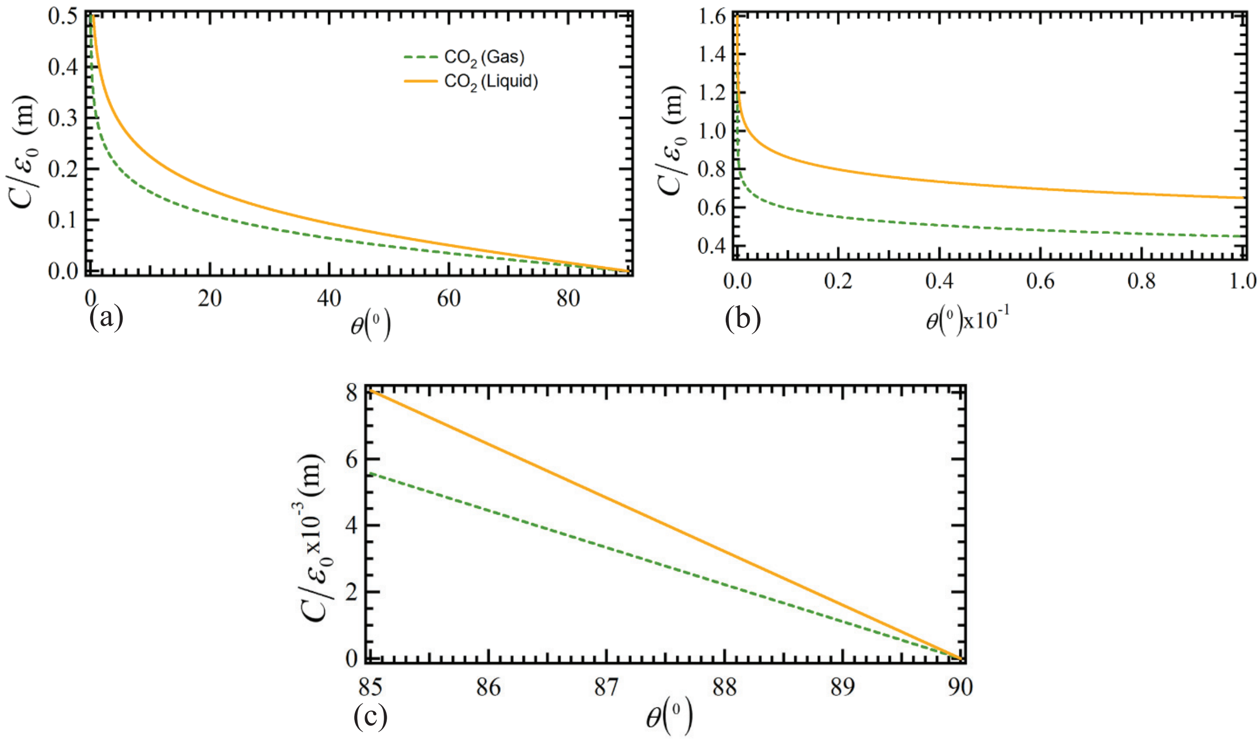

FIGURE 3. (a) The Capacitance of the SCS related with the variation of θ including with the variation of dielectric materials. (b) The Capacitance of the SCS related with the variation of θ including with the variation of dielectric materials for the low angle. (c) The Capacitance of the SCS related with the variation of θ including with the variation of dielectric materials for the large angle.

Figure (3) shows the capacitance of the SCS related to the variation of θ including the variation of dielectric materials inside the cavity. In this work we use carbon dioxide as dielectric materials inserted into the cavity of the SCS. We use carbon dioxide as liquid $(\kappa = 1.4492)$ and gas $(\kappa = 1.000922)$ respectively[11]. The two different phases of carbon dioxide representing fluid, in which consist of liquid and gas phases. Figure (3a) shows that by inserting dielectric material will be increasing the capacitance of the SCS. This is similar to a general case of a capacitor, in which by inserting dielectric materials will increase capacitance. The dielectric materials produce internal electric field caused by polarization of dielectric molecules. The internal electric field partially cancels the electric field from the capacitor plates. This reduces the net electric field inside the capacitor in which reduce the voltage to the capacitor plates. Extra charges are needed to restore this conditions. The net electric field effect is causing increase a capacitance of the SCS.

Figure (3b) shows that the capacitance of the SCS with dielectric materials has reached the maximum for the low value of θ. For same dielectric materials, decreasing value of θ related with increasing area of the capacitor plates has not significantly affect with the capacitance. The SCS has reached a maximum value of the capacitance. For the low angle, dielectric materials have an important role in increasing the capacitance of the SCS. The dielectric material with a large value of a dielectric constant can hold more charge in which can increase the capacitance.

Figure (3c) shows that the capacitance of the SCS with dielectric materials at large value of θ. At large angle, we show that the SCS inserted with materials with a large dielectric constant has decreased significantly related with increasing of the θ. In this case either the dielectric materials or θ has significant rule to the capacitance of the SCS.

CONCLUSION

In conclusion, we have reported the analytical solutions of capacitance for the SCS. The capacitance of the SCS depends on angle between the SCS plates (θ) and the length of the SCS (z), in which related to the area of the SCS. We show that the capacitance of the SCS increase along with the decreasing value of θ and increasing the value of z, in which related with the increase of the SCS plates. Otherwise, the capacitance of the SCS decrease along with increasing value θ and decreasing value z. The capacitance of the SCS increase by inserting dielectric materials. At a low value of θ dielectric materials have a significant rule to increase the capacitance of the SCS, in which the capacitance has reached the maximum capacity to hold a charge. At a large value of θ we show that the capacitance of the SCS depends with θ and the dielectric materials. The SCS inserted with dielectric material with large dielectric constant has decreased significantly related with increasing value of θ.

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